Structure analysis of molecular clouds: Observations and simulations

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The self-similarity of molecular cloud structure has to be taken into account when analyzing cloud maps and interpreting them in terms of physical parameters. We have used the Δ-variance analysis and the area-perimeter relation of iso-intensity contours as measures for the complex density structure. Detailed investigations of Polaris flare maps show that the cloud structure can be interpreted as fractional Brownian motion fractal. Radiative transfer simulations on fBm density structures enable us to fit all observed map characteristics but dismantled uncertainties regarding the velocity structure which can be solved only by the help of further high S/N observations.

1. Introduction

Molecular cloud observations with varying spatial resolutions reveal a complex, often self-similar structure over a wide range of scales. This has to be considered in the analysis and simulation of molecular clouds. Assuming the simple picture of homogeneous clouds or non-penetrating clumps in a homogeneous interclump medium will necessarily fail.

More appropriate methods include the analysis of the power spectrum of density fluctuations (e.g. Green 1993), computations of the fractal dimension (e.g. Vogelaar & Wakker 1994) or multifractal studies (e.g. Brunt 1998). However, until now these concepts are only applied to analyze the structure of the sky projected cloud images and not to derive the physical parameters of the underlying 3-D cloud structure. To determine the characteristic parameters including the density, temperature, and velocity distribution, the methods have to be combined with radiation transfer computations providing the line excitation temperatures and the emergent line profiles and intensities.

In advanced cloud models, the density structure needs to fulfil the scaling laws found for the structure in the observed 2-D spectral line maps. They have to reproduce the spatial correlations in the maps as well as the observed line ratios and profiles. As a first approach, we have set up a simple fractal cloud model. Using fractional Brownian motion (fBm) fractals we can consistently fit the scaling laws for the observed structures and the characteristics of the line profiles in the observed transitions.

2. Analysis methods

We have used two different methods to study the observed cloud maps: the Δ-variance analysis and the determination of the area-perimeter relation of iso-intensity contours. The results served as a starting point for the fractal cloud models. The methods are also applied to the simulated maps for comparing their characteristics with the observations.

The Δ-variance was introduced by Stutzki *et al.* (1998) as an n-dimensional generalization of the Allan variance. It contains all the information from the power spectrum of the images but is better suitable to determine the power spectral index of the observed cloud structure, since it allows a better separation from noise, beam smearing, and baseline effects. For a power spectrum of a 2-D image following $P(f) \propto f^{-\beta}$, the Δ-variance is a power law: $\sigma_\Delta^2(r) \propto r^\alpha$ with $\alpha = \beta - 2$ ($\alpha = \beta - n$ for n-dimensional structures).

The area-perimeter relation as a measure of the fractal dimension of the map structures was extensively tested by Vogelaar & Wakker (1994). For fBm structures they found a fixed relation between the power spectral index β, the fractal box-counting dimension d, and the exponent of the area-perimeter relation s: $s = (d-1)/2$ and $\beta = 8 - 2d$ for 2-D images. Repeating their tests, we found deviations for fBms with β close to 2 and 4. An additional disadvantage of the analysis in terms of the area-perimeter relation compared to the Δ-variance is the lower reliability of the results in case of noisy data.

fBm structures can serve as a simple test case since the relations between the different parameters are well known for these fractals. fBms are completely determined by a power law power spectrum with index β and random Fourier phases (Peitgen & Saupe 1988).

3. Observations

For our analysis we used CO observations of MCLD 123.5+24.9, a high latitude cloud in the Polaris Flare. The cloud was observed at high angular resolution as part of the IRAM key project "Small-scale structure of pre-star-forming regions" (Falgarone *et al.* 1998). Complementary observations at lower angular resolution were available from the KOSMA 3m telescope and the CfA 1.2m telescope (Heithausen & Thaddeus 1990).

The IRAM 30m observations cover the dense core and extend over a large area of its environment, including the nearest cloud edge. Maps were taken in ^{12}CO and ^{13}CO 1–0 and 2–1 and in the ^{18}CO 1–0 transitions. They are fully sampled and contain 2200 up to 3200 spectra with a velocity resolution of 0.05 km/s and good signal to noise.

Fig. 1 shows the ^{13}CO and ^{12}CO 1–0 integrated IRAM maps together with the line profiles at five positions and the average line profile in both transitions. Strong discrepancies between the maps in ^{12}CO and ^{13}CO are obvious. The ^{13}CO lines are considerably narrower than the ^{12}CO lines throughout the whole cloud. Various positions in the map show moderately flat ^{12}CO profiles indicating line thermalization and saturation.

The application of the Δ-variance analysis to the integrated and channel maps in all five transitions shows a power law power spectrum and random phases in Fourier space for most maps. The index of the Δ-variance α for the maps in the optically thin molecules is about 1.4 corresponding to a power spectral index $\beta \approx 3.4$. In the optically thick ^{12}CO IRAM maps $\alpha \approx 0.8$ corresponding to $\beta \approx 2.8$. There is no significant difference in the spectral indices of the line integrated maps and individual channel maps.

The area-perimeter analysis of iso-intensity contours in the maps provides indices $s \approx 0.63$ and $s \approx 0.76$ for the IRAM ^{13}CO and the CfA ^{12}CO maps, respectively. These values correspond to $\beta \approx 3.5$ and $\beta \approx 3.0$, close to the results from the Δ-variance analysis. In ^{12}CO, we had to use the larger CfA maps for the area-perimeter analysis because the IRAM maps did not contain enough closed contours. The results from both methods show that the observed CO maps are consistent with 2-D fBm structures.

4. Fractal cloud models

Motivated by these results, we have set up a 3-D fBm fractal model for the cloud density structure. Basing on the fact that the projection of a 3-D fBm with a spectral index β is an fBm image with the same index (cf. Stutzki *et al.* 1998), it is reasonable to assume a 3-D fBm density structure with the same spectral index as found for the optically thin maps.

4.1. *Simulations*

The fBm fractals were produced via Fourier transform algorithm on a 128^3 cubic grid. In an iterative procedure, the line excitation rates are computed for each density point.

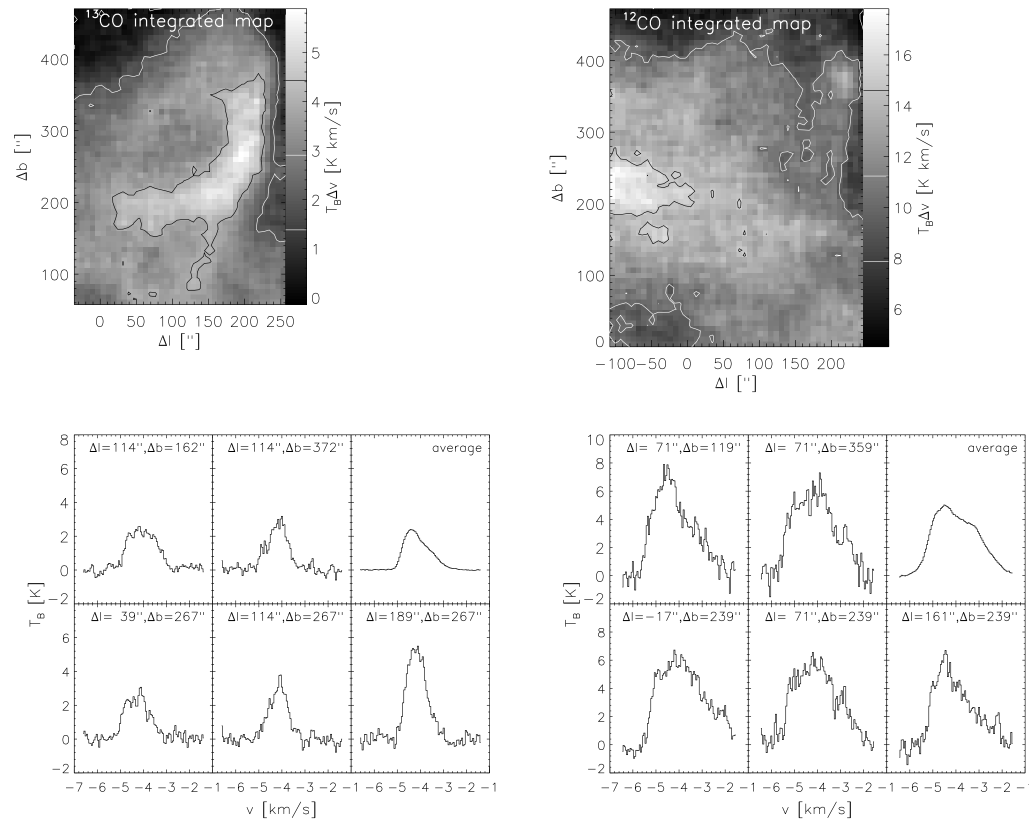

FIGURE 1. Integrated line maps of the ^{13}CO and ^{12}CO 1–0 transitions, observed towards MCLD 123.5+24.9 (upper panels). The lower panels show the line profiles at 5 positions on a cross cut together with the average spectrum. The left column corresponds to ^{13}CO, the right column to ^{12}CO.

We use two simplifications: locally, the level populations are computed by means of the Sobolev approximation exploiting the strong velocity variations in the cloud (Ossenkopf 1997). The non-local interaction is taken into account by the iterative computation of the average line intensity within the cloud that is used as external field in the Sobolev approach.

To obtain spectral line maps comparable to the observations, a ray-tracing technique combined with a Gaussian beam convolution is applied and white noise was added. Several isotopes and transitions are simulated and the different model parameters are varied to fit the observations.

4.2. *Physical effects in radiative transfer*

Most large maps are obtained in ^{12}CO 1–0 and 2–1 rotational transitions. However, these transitions are often optically thick and the line intensities do not exactly trace the density structure. We tested how optical depth effects change the slope of the Δ-variance and the area-perimeter relation compared to the parameters of the original density structure. It turns out that optically thick lines produce always maps characterized by a spectral index β lower than that of the original density structure. The results are the same whether we measure β via the Δ-variance or the area-perimeter relation. A turn-over in the Δ-variance at large lags often provides a better signature for line saturation than the simple inspection of the line profiles.

A main uncertainty in the radiative transfer computations is the cloud velocity structure. The relation between the density and the velocity structure in a molecular cloud is still poorly understood. Hydrodynamic computations over a broad range of length scales may help to solve this problem in the near future. Here, we have tested five simple heuristic velocity models. We assumed that either the velocity correlation length or the velocity dispersion is related to the local density or the local density gradient. In the last model the velocity structure is represented by a second fBm. We found that none of these simple models fulfils all of the constraints provided by the observations. Models with velocity structures independent from the density can be ruled out. A combination of the first four approaches gave the best fit to the observations. A quantitative analysis of the shape of the line wings form high S/N observations can help to discriminate between different models.

Another inherent observational problem is the finite resolution of the telescope beam and the limited signal-to-noise ratio. Our model simulations of these effects show that the finite beam size does hardly affect the resulting fractal parameters of the observed images as long as the maps are large enough. From the smoothness of the line profile, one can however deduce the size of single clumps (or discretization units) compared to the beam size as discussed already by Tauber (1996). This requires very high S/N ratios. In case of a low signal to noise, only the Δ-variance provides the correct spectral indices of the maps.

5. Fit of the observations

Using the simple fractal model we find a reasonable fit to the Polaris Flare observations with respect to the line strengths, widths, and ratios in all transitions, the general line shape and the spatial properties of the emission characterized by the Δ-variance and the area-perimeter relation.

The only deviation we got was the average ratio between the 2–1 and 1–0 lines which was given in the observational data to be 0.7 for ^{13}CO and 0.6 for ^{12}CO. We could not fit this simultaneously with a cold cloud model. We obtain a ratio of 0.5 in ^{13}CO when reproducing the ^{12}CO ratio.

We found several "best fit" models since some parameters are not sufficiently constrained by the observations. Thus, the size of single clumps where the fractal scaling breaks down can be $0.001 - 0.004$ pc, the average gas density is $n_{\rm H_2} = 2.0 - 4.0\,10^3$ cm^{-3} and the average column density $N_{\rm H_2} = 0.5 - 1.6\,10^{21}$ cm^{-2}. Better constrained parameters are the gas temperature $T_{\rm gas} = 10$ K and the total velocity dispersion in the cloud $\Delta v = 1.65$ km/s. Fig. 2 shows the maps from one of these fits to be compared with the observations in Fig. 1.

6. Conclusions

We found that the Δ-variance in the integrated maps of optically thin molecules is a useful tool to determine the power spectrum of the cloud density structure. Moreover, the Δ-variance is very sensitive to the optical depth and the velocity structure. Fractal models with a continuous density structure represent a promising and simple approach to explain the observed spectra and maps of molecular clouds. The fingerprints of the density structure are also contained in the differences between the line profiles of different isotopes. There must be a tight relation between the density and the velocity structure

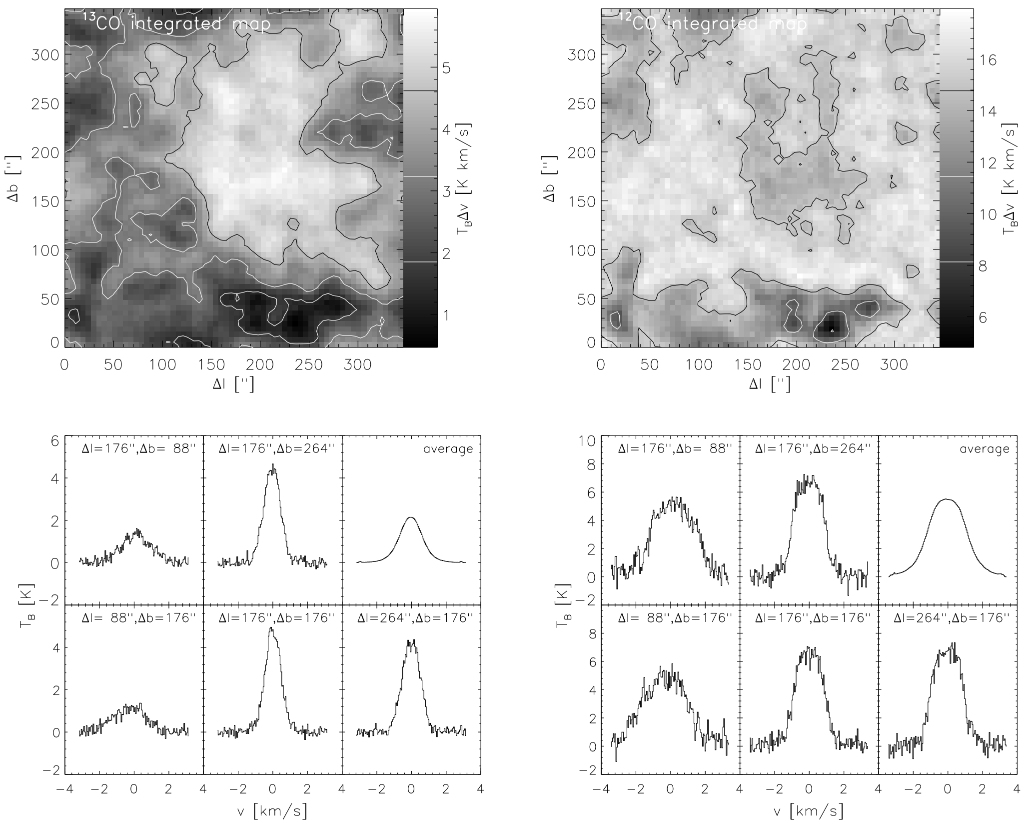

FIGURE 2. Simulated maps and line profiles of the ^{13}CO and ^{12}CO 1–0 transitions from a model fit to the MCLD 123.5+49.9 observations. The panels are the same as in Fig. 1.

in the cloud. Hydrodynamic computations are necessary to complement our empirical results. Further high S/N observations can help to clarify the cloud density and velocity structure.

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