Measuring the structure of turbulent clouds

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Why to measure the cloud structure?

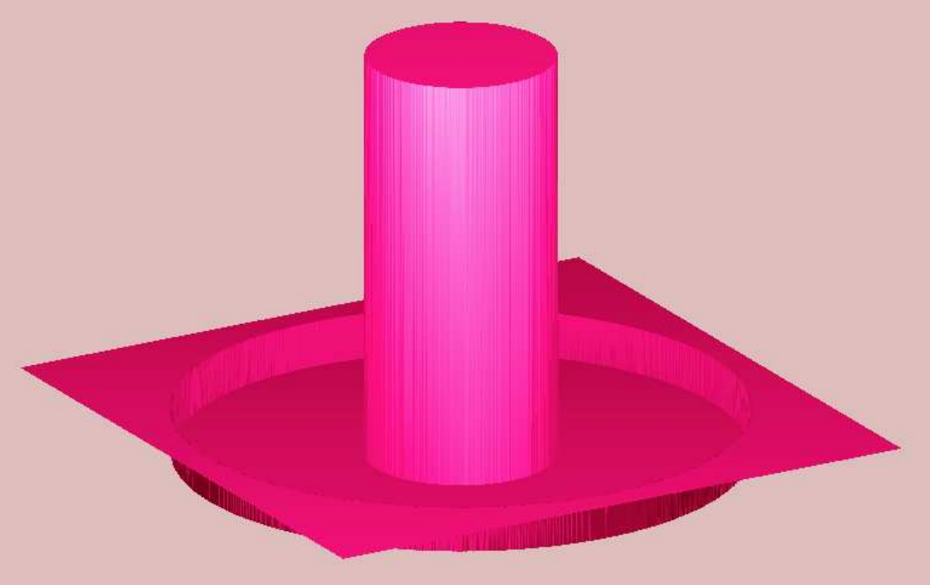
Molecular clouds show a hierarchy of complex structure which can be attributed to interstellar turbulence. To study the dynamic nature of molecular cloud turbulence we have to compare the statistical properties of molecular cloud observations with hydrodynamic and magnetohydrodynamic turbulence simulations.

There is a large variety of statistical measures to characterise the scaling of the density and the velocity structure. We have tested these tools with respect to their sensitivity and applicability to molecular line data and turbulence models. The most significant measures are applied in the comparison of observational data with numerous turbulence models to reveal the dynamical state of the molecular clouds.

How to measure the cloud structure?

The Δ-variance

The Δ-variance (Stutzki et al. 1998) measures the relative structural variation on a certain scale. It can be used to determine the spatial scaling properties of arbitrary quantities, e.g. intensity maps or velocity centroid maps.



The Δ-variance is computed by convolving the image with a "French hat" wavelet of varying diameter and measuring the variance of the resulting map.

The slope α of the Δ-variance as a function of lag (filter size) is related to the spectral index β of the power spectrum of the image

$$P(|\vec{k}|) \propto |\vec{k}|^{-\beta} \quad \text{by} \quad \beta = \alpha + 2$$

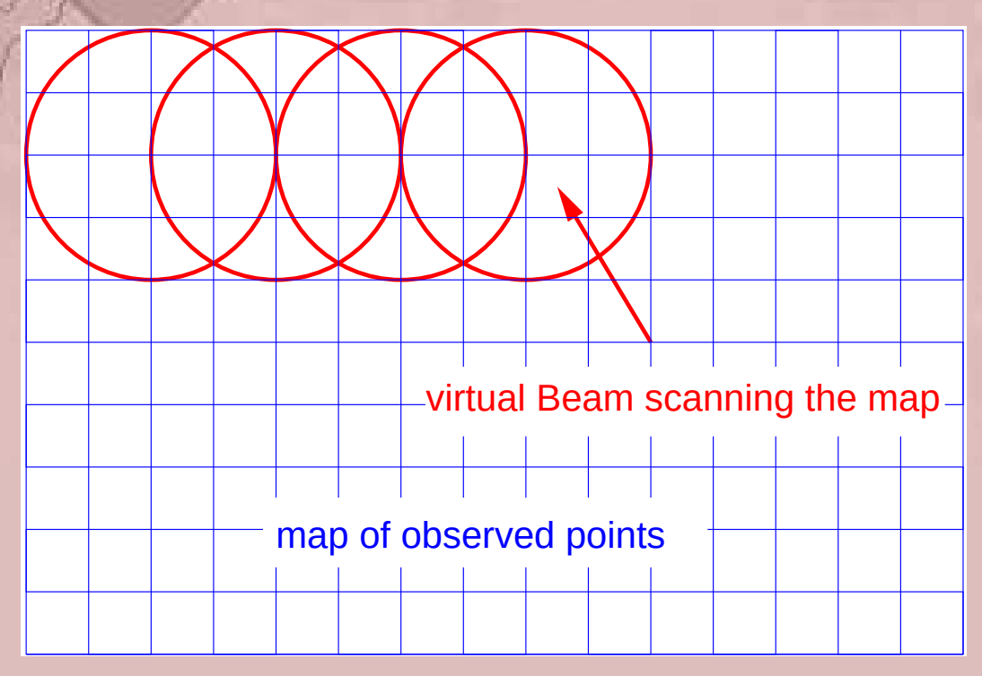
The Δ-variance measures the spectral index in a way which is more robust with respect to edge and gridding effects than the Fourier transform.

Velocity scaling in the Δ-variance

Like for intensity maps the analysis may as well be applied to the maps of velocity centroids, i.e. the first moments of the line profile. The square root of the Δ-variance then gives the spatial drift of the velocity structure in terms of a variation of the line position.

The size-linewidth relation

The traditional size-linewidth relation (Goodman et al. 1998) relies on the selection of discrete objects (clumps or clouds) which are impossible to define in filamentary turbulent structures. We use a scanning-beam size-linewidth relation to trace different length scales.



The map is scanned by virtual telescope beams of varying size. We compute the average velocity variation measured within a beam depending on its size.

Here one can consider either the total line width in the virtual beam, tracing the full velocity distribution, or the distribution of centroid velocities only. In the first case all components along a line of sight contribute. In the latter case only the lateral variation of the velocities is measured. The comparison of both functions can reveal the depth of the cloud along the line of sight.

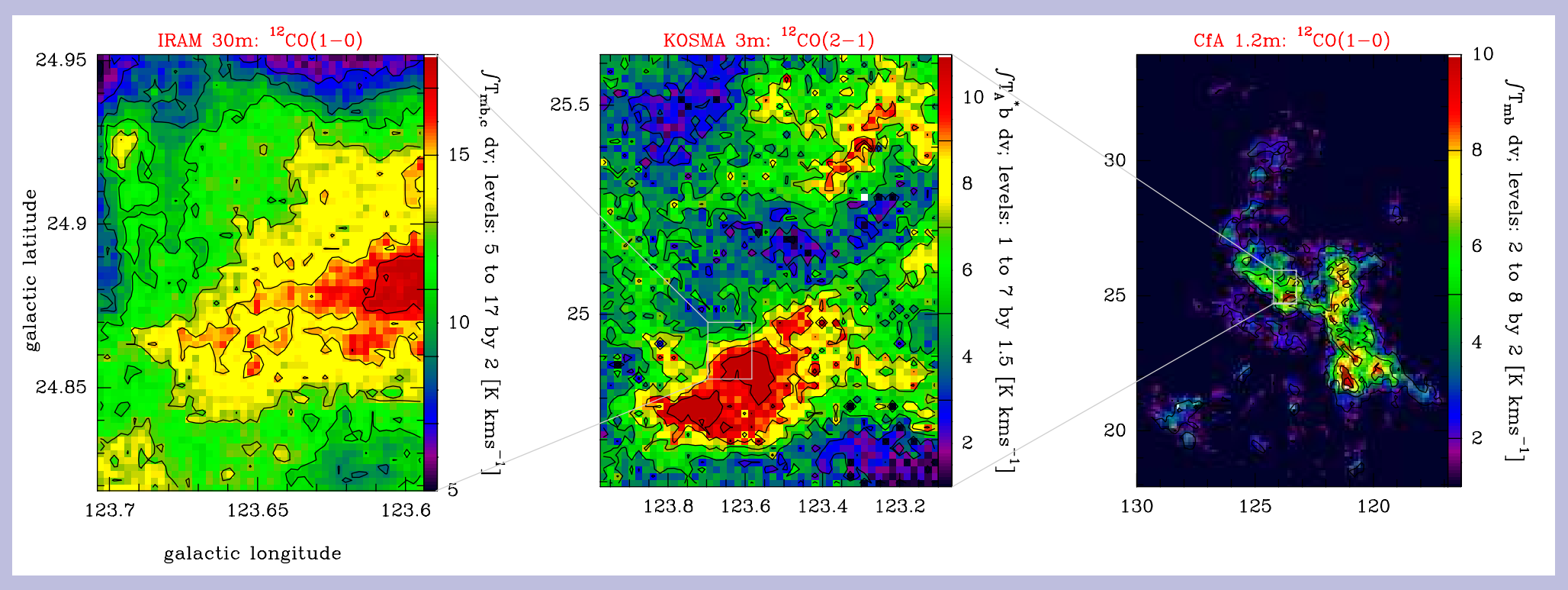
PDFs of line-of-sight velocities

The velocity probability distribution functions (PDFs) are clues to the character of turbulence. Exponential wings indicate intermittency as produced by strong stellar winds, whereas Gaussian tails characterise random flows and decaying turbulence models.

The velocity PDFs are either measured as average line profiles tracing the full velocity distribution (Falgarone & Phillips 1990) or as the distribution of velocity centroids (Miesch & Bally 1994). In the first case all components along a line of sight contribute. As for the size-linewidth relation the comparison of both PDFs can reveal the cloud extend along the line of sight.

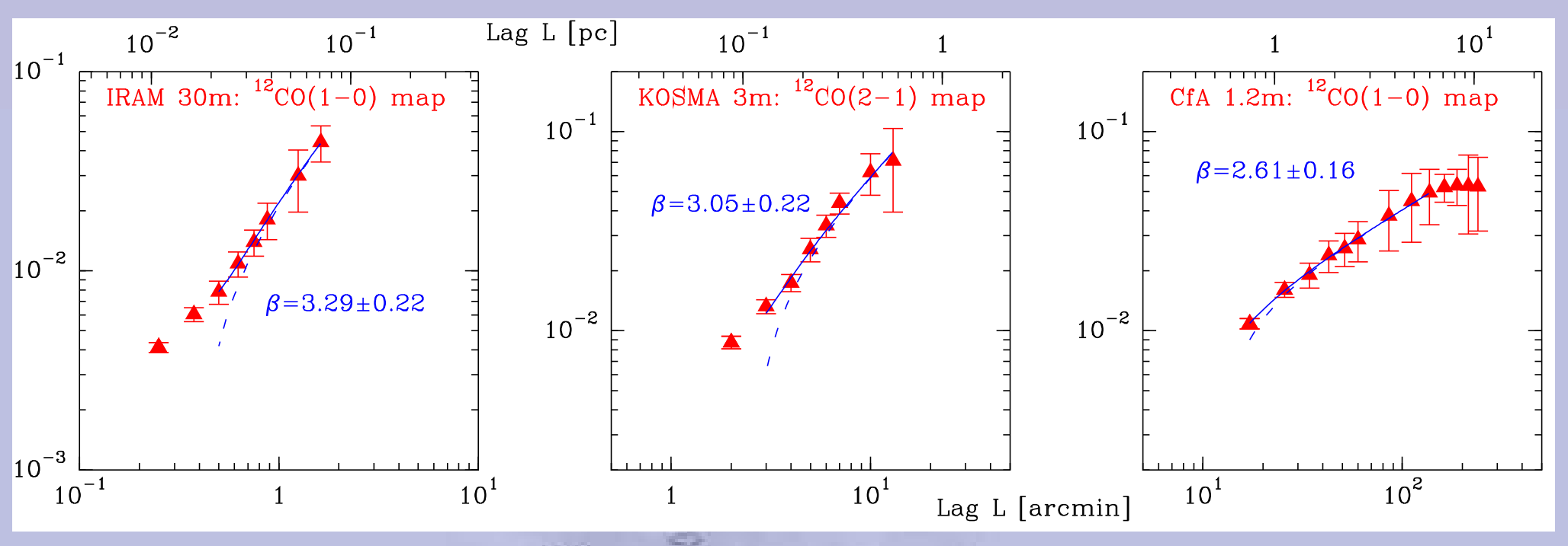
Cloud observations

- We have analysed extended CO maps of various molecular cloud complexes with different star-formation activity.
- A preferred example is MCLD 123.5+24.9 in the Polaris Flare, a high latitude cloud without star formation, where a combination of observations in different CO isotopes and with different resolutions is available. They cover a large dynamic range of spatial scales.



CO maps of three nested data cubes of MCLD 123.5+24.9 observed with the IRAM 30m telescope (Falgarone et al. 1998), the KOSMA 3m telescope (Bensch et al. 2001), and the CfA 1.2m telescope (Heithausen & Thaddeus 1990).

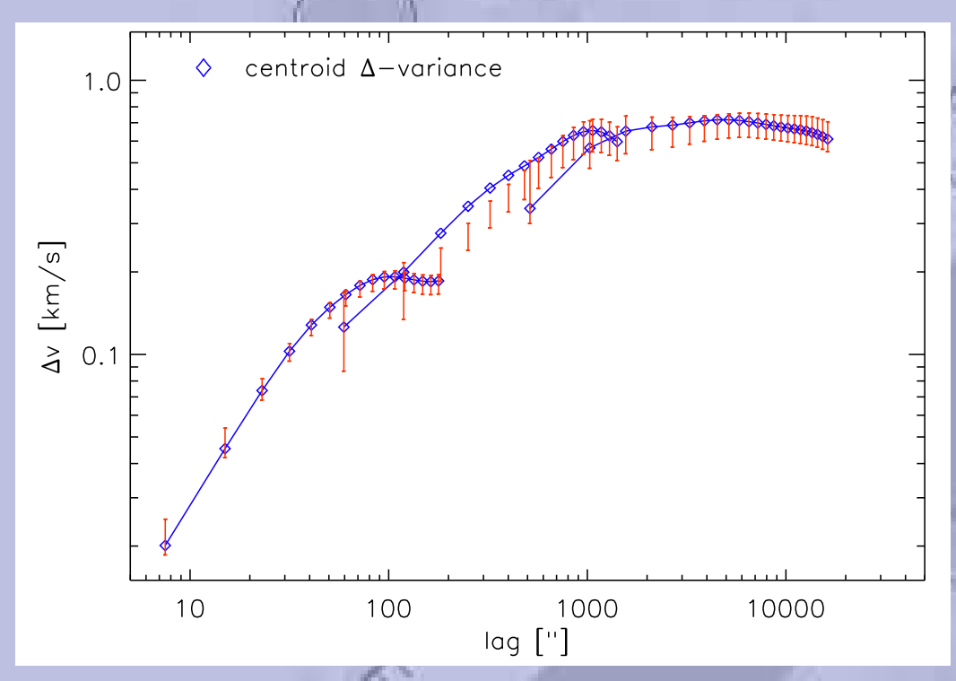
Δ-variance of the intensity maps



The Δ-variance σ_Δ^2 of the three nested MCLD 123.5+24.9 maps. The spectral indices β correspond to the solid lines including corrections for noise and beam smearing

On the scale of a typical map most observations are reasonably fitted by a power law. There is only one systematic trend visible: the spectral index β decreases when increasing the absolute length scale mapped.

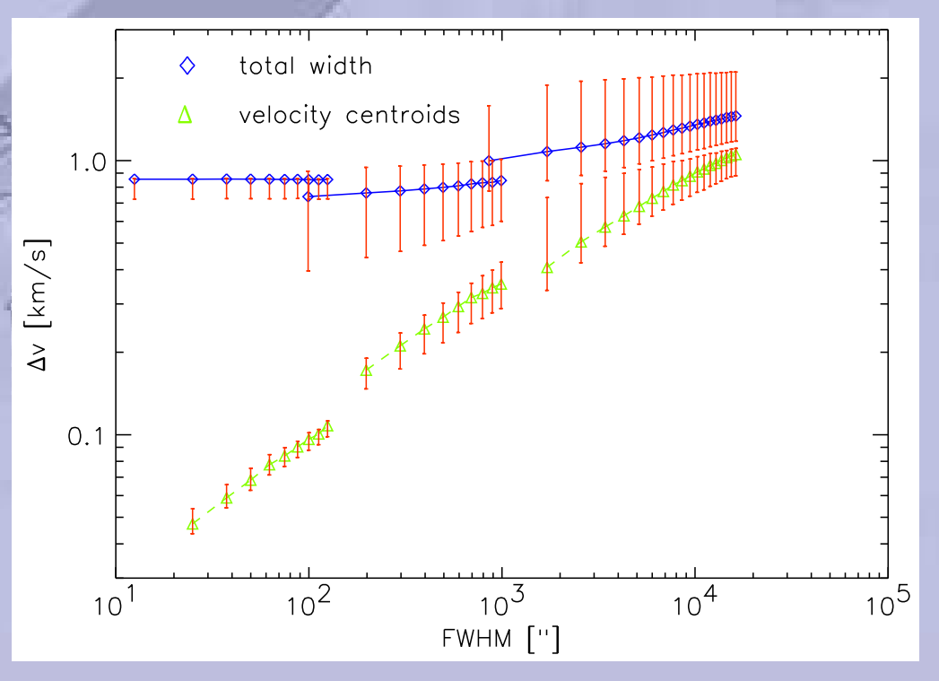
Δ-variance of the centroid velocities



Δ-variance of the MCLD 123.5+24.9 velocity centroids. The connecting lines represent the IRAM, the KOSMA, and the CfA maps on their respective scale. Beyond 3000″ the centroids are dominated by regions without emission so that the values are not significant there.

We find one smooth relation for the velocity drift connecting all three maps. The spectral index β changes from 4.3 at the smallest scales to 3.9 at the end of the significant range.

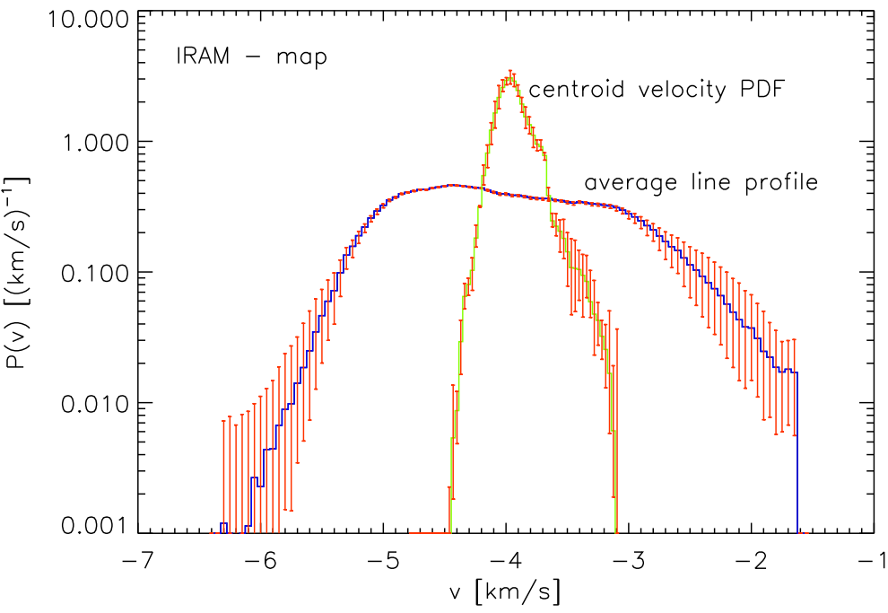
The size-linewidth relation



Size-linewidth relations for the MCLD 123.5+24.9 data. The three maps provide the three connected size ranges.

The size-linewidth relation for the centroid velocities can detect the spatial velocity drift on all scales. We find a power law index of 0.5 covering all scales. Comparing it with the relation for the total line profiles we estimate a cloud depth corresponding to about 150′ or 6 pc respectively.

The velocity PDF

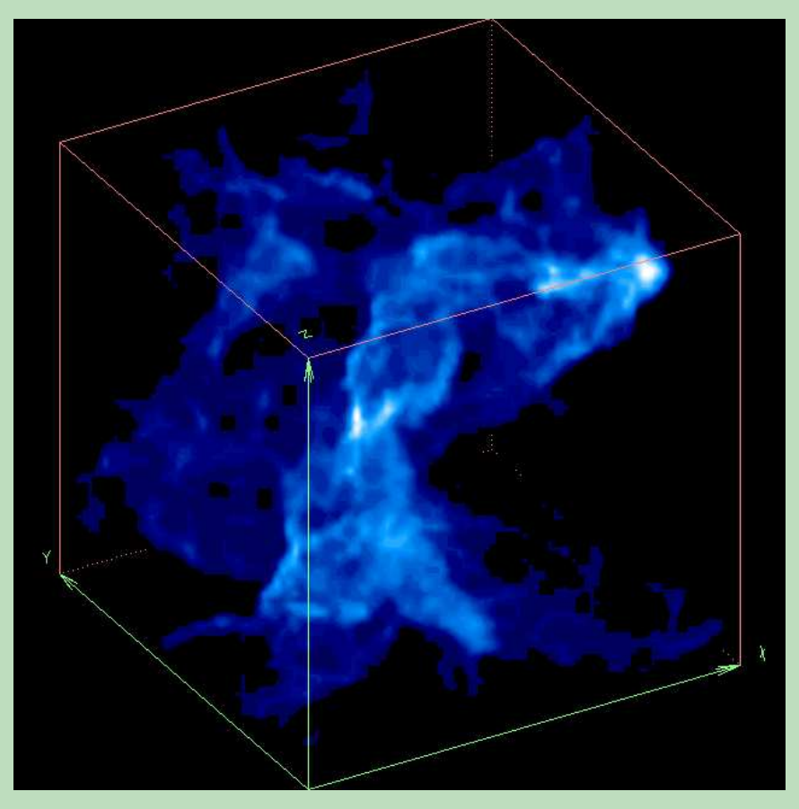


Total velocity PDFs as measured by the average line profile and centroid velocity PDFs for the IRAM data. The error bars are provided by the possible influence of observational noise.

The PDFs are well represented by the superposition of two Gaussians with a width ratio of about 2.
Only when the map size is comparable to the depth of the cloud, the total velocity PDF and the centroid velocity PDF agree.

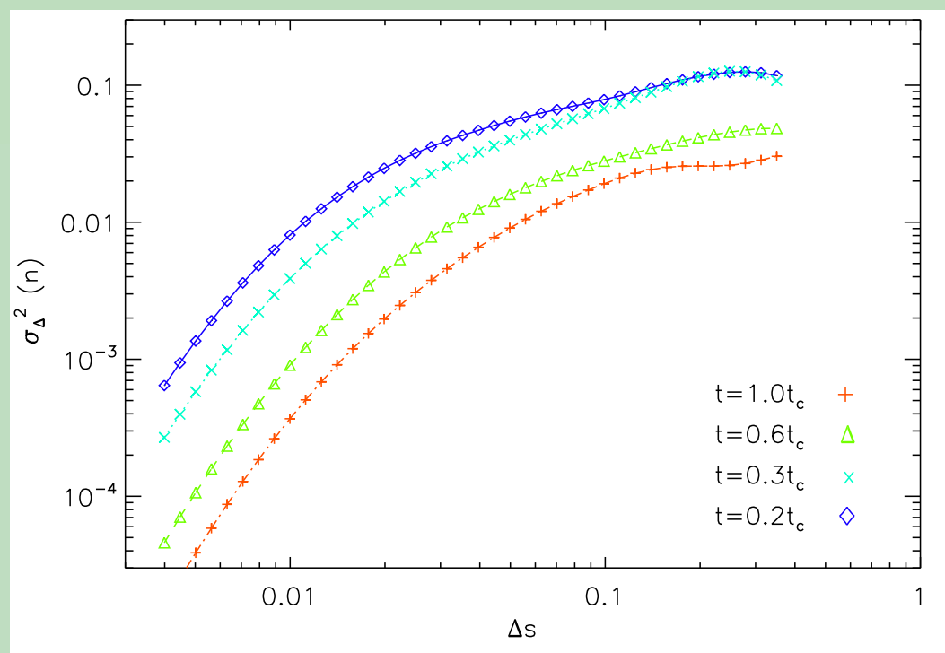
Cloud simulations

- hydrodynamic or magnetohydrodynamic models
- ZEUS-3D code on a $64^3 - 256^3$ grid or SPH code with up to $5\,10^5$ particles (Stone & Norman 1992, Mac Low et al. 1998, Klessen 2000)
- isothermal, supersonic turbulence
- driven by Gaussian perturbations on a fixed scale or a range of scales
- continuously driven models or decaying turbulence
- gravitational collapse



Density structure in a hydrodynamic turbulence model driven at large scales before the formation of the first gravitationally collapsing cores.

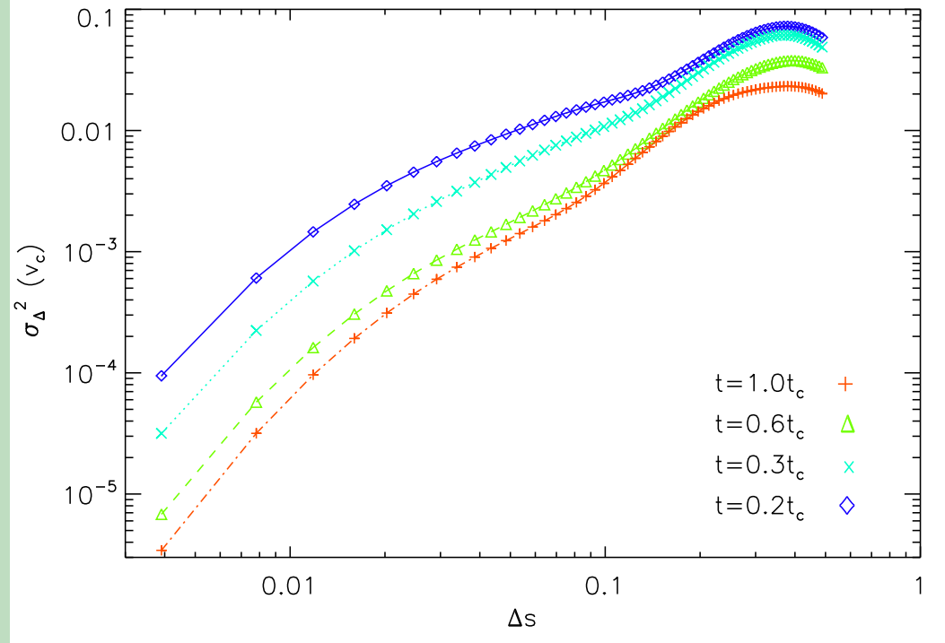
Δ-variance of the column density



Time sequence of Δ-variances for a model of decaying hydrodynamic turbulence where the energy was initially injected at the largest scale.

The turbulent cascade appears as power-law below a peak at the scale of the energy input. Below $\Delta s < 0.02$ structure is blurred by numerical dissipation. The spectral index β varies between 2.4 to 2.7 in the inertial range and 4.0 at the dissipation scale. The decay of turbulence removes structure from the small-scale end leading to a curved Δ-variance plot.

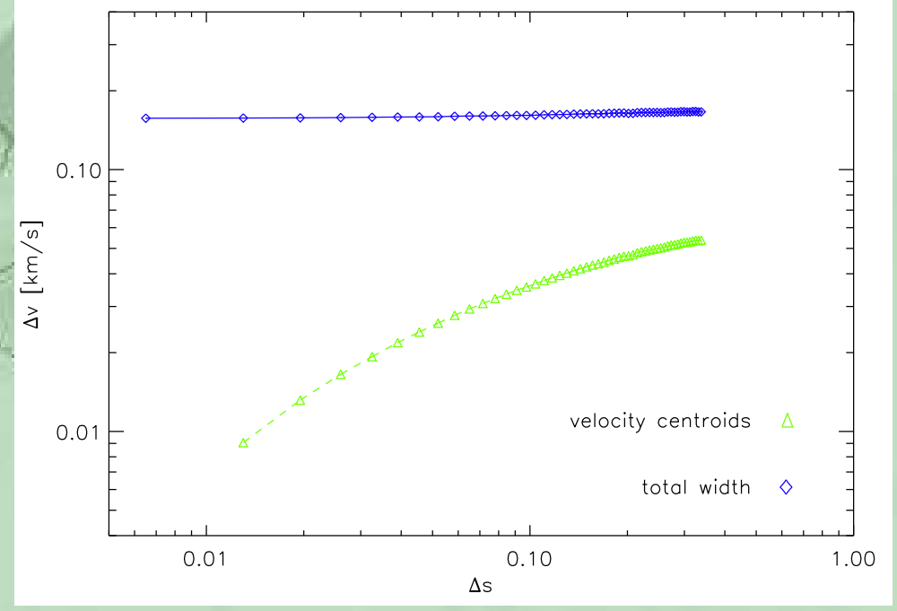
Δ-variance of the centroid velocities



Δ-variance of the centroid velocity maps for the decaying hydrodynamic model shown above.

The velocity structure shows the same general behaviour as the density structure but the dissipation scale appears less pronounced. The spectral index β changes from 3.6 at large scales to 4.2 at the smallest scales.

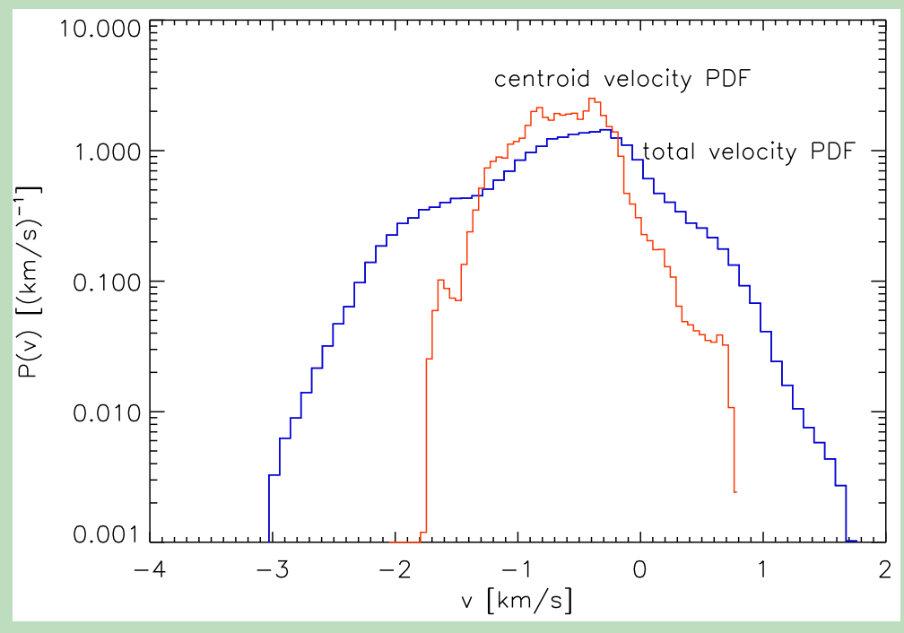
The size-linewidth relation



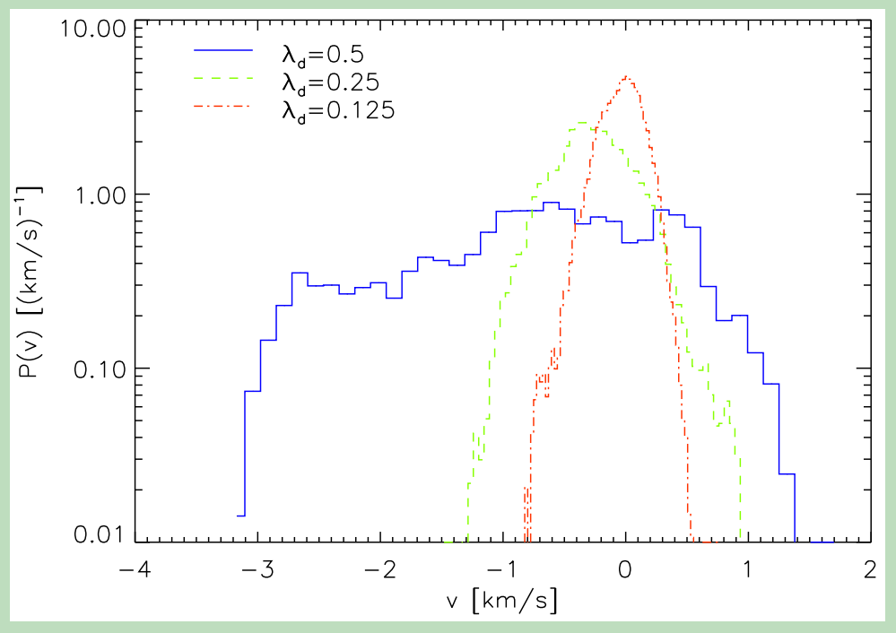
Size-linewidth relations for the initial step of the decaying hydrodynamic model.

For most models the size-line width relation can be approximated by a power law with an exponent between 0.4 and 0.6 independent from the evolutionary state or the magnetic field. Above the driving scale we find a flattening of the relation that should be detectable in corresponding observational data.

The velocity PDF



The total velocity PDF and the centroid velocity PDF for a driven hydrodynamic model.



Centroid velocity PDFs of hydrodynamic models driven at three different length scales.

Large scale driven models have irregular PDFs because of the small number of modes. Models driven with smaller eddies show close to Gaussian wings for all simulated scenarios.

Conclusions

- The Δ-variance is the most sensitive measure for the density and velocity scaling behaviour in molecular clouds.
- The Δ-variance shows that the Polaris Flare is not self-similar, both in the intensity and the velocity structure. This is only detectable when combining maps taken at different resolutions covering a large dynamic range.
- The width ratios of the two different velocity PDFs is a measure for the depth of a cloud along the line of sight. The observed Gaussian wings provide no indication of strong intermittency in the molecular cloud turbulence.
- The observed Δ-variance behaviour excludes several turbulence scenarios:
  - Only large-scale driven turbulence models fit.
  - Strong magnetic fields are excluded.
  - The driving may have stopped only recently.
- The observed steepening of the Δ-variance at small scales resembles the effect of numerical dissipation in the models. This may indicate ambipolar diffusion (Zweibel & Josafatsson 1983).
- No model fits all characteristics of the observed structure yet.

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